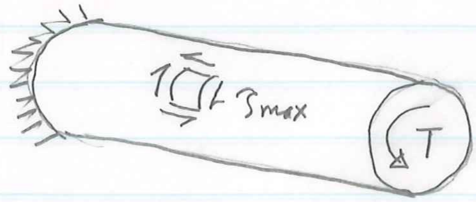
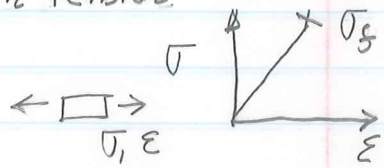
12. Ultimate value of torque T for solid circular shaft (homogeneous, isotropic material)

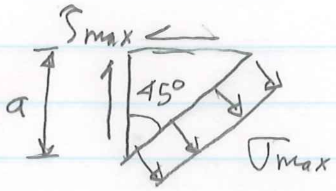
Consider a brittle material that fractures in tension.



radius c



$f \equiv$ fracture



$$\sigma_{max} \sqrt{2}\, a = 2\, \tau_{max} \cdot a \cdot \frac{\sqrt{2}}{2}$$

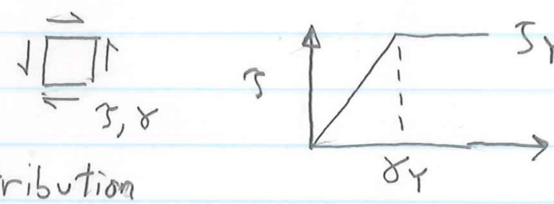
$$\sigma_{max} = \tau_{max}$$

$$T = \tau_{max} \cdot J/c = \sigma_{max} \cdot J/c$$

at fracture, $\sigma_{max} = \sigma_f$

so $T_f = \sigma_f \cdot J/c$

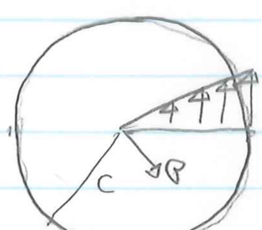
Consider a ductile material with elastic-plastic behavior



$Y \equiv$ yield

Examine the shear stress distribution on a cross-section as T increases (states 1, 2 and 3).

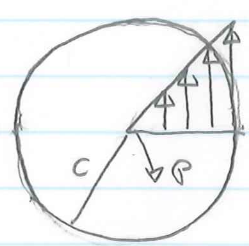
1.



$\tau_{max} < \tau_Y$

$T = \tau_{max} J/c$

2.

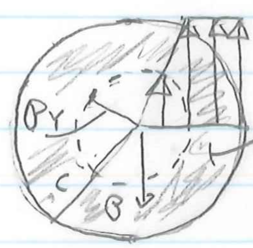


$\tau_{max} = \tau_Y$

$T = \tau_Y J/c = T_Y$

T_Y = Torque at which yielding first occurs

3.



$\tau_{max} = \tau_Y$

yielded zone for $\rho \geq \rho_Y$

$T_Y < T < T_P$

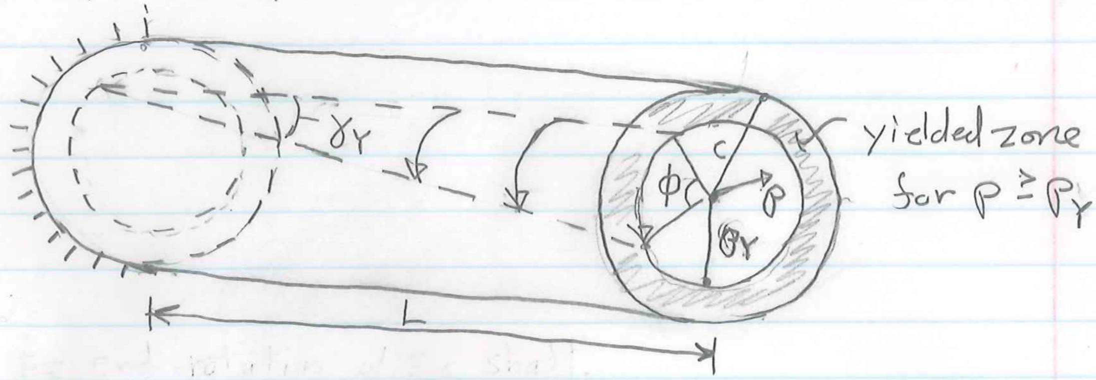
$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi \left[\frac{\tau_Y}{\rho_Y} \int_0^{\rho_Y} \rho^3 d\rho + \tau_Y \int_{\rho_Y}^{c} \rho^2 d\rho \right]$$

$$= \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_Y^3}{c^3}\right) \rightarrow T_P \text{ as } \rho_Y \rightarrow 0$$

where T_P is the torque when the cross-section is fully yielded (plastic torque).

$$T_P = \frac{4}{3} T_Y$$

The shear strain γ is still a linear function of ρ, which means each cross-section still rotates as a rigid planar section. Consider the twisted shaft under a torque T where $T_Y < T < T_P$ (state 3) on previous page.



ϕ is the end rotation of the shaft. From geometry,

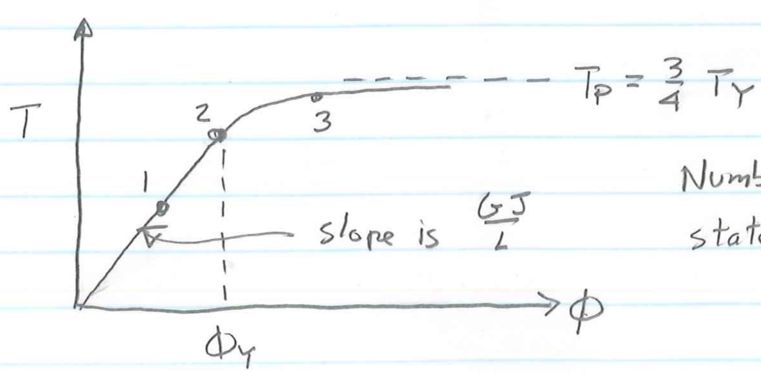
$\gamma_Y \cdot L = \phi \cdot \rho_Y$. Define ϕ_Y as the end rotation when yielding first occurs, ie, when $\rho_Y = c$. Then,

$\gamma_Y \cdot L = \phi_Y \cdot c$. Thus, $\dfrac{\rho_Y}{c} = \dfrac{\phi_Y}{\phi}$

This expression can be substituted into the equation for T in state 3:

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\phi_Y^3}{\phi^3}\right)$$

T can now be plotted as a function of end rotation ϕ.



Numbers correspond to states on previous page.